Three-flavor analysis of long-baseline experiments

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We compare the analysis of existing and future neutrino oscillation long-baseline experiments, where we point out that the analysis of future experiments actually implies a 12-dimensional parameter space. Within the three-flavor neutrino oscillation framework, six of these parameters are the fit parameters, and six are the simulated parameters. This high-dimensional parameter space requires the condensation of information and the definition of performance indicators for the purpose needed. As the most sophisticated example for such an indicator, we choose the precision of the leptonic CP phase, and discuss some of the complications of its computation and interpretation.

Long-baseline experiments, such as conventional beams, superbeams, neutrino factories, or even new reactor experiments, are just being started with the K2K accelerator-based experiment [1]. Because of the complicated intrinsic structure of the appearance channels in accelerator-based experiments, correlations [2] and degeneracies [3,4,5,6] play a major role in the analysis of these experiments. In this talk, we refer to "correlations" as connected degenerate solutions (at the chosen confidence level), and to "degeneracies" as disconnected degenerate solutions (at the chosen confidence level). The correlations and degeneracies appear in any *fit* manifold, such as a fit to the data. They come from the intrinsic structure of the oscillation probabilities, which implies that an experiment cannot uniquely resolve the individual parameters. In the χ^2-approach, they lead to the final precision of the quantity of interest by projection of the fit manifold onto the respective parameter axis (or plane).

There is, however, one important difference between the analysis of existing and the simulation of future experiments: For existing experiments, the data are provided by the experiment, whereas for future experiments, the data have to be simulated. Thus, the topology of the fit manifold does, for future experiments, not only depend on the fit parameter values, but also on the simulated parameter values. The interpretation of such simulated parameter values is as follows: "If the actual value, which nature provides, corresponds to a certain simulated parameter values, then the measurement performance will be ...". Therefore, one actually faces a 6+6-dimensional parameter space if one simulates a future experiment. Fortunately, the potential simulated parameter values are not entirely free, since some of them have been already measured to certain precisions. For example, the leading solar and atmospheric parameters are quite precisely known. Nevertheless, it turns out that the simulated value of Δm^2_{31} has a rather large impact on the potential of future accelerator-based experiments. In addition, the simulated values of the mass hierarchy, $\sin^2 2\theta_{13}$, and δ_{CP} strongly influence the respective measurements.

Because of this high-dimensional parameter space, it is often convenient to define performance indicators which condense the information. The purpose of these indicators can be risk minimization with respect to the simulated parameter values, optimization, or the comparison of different strategies. Once such indicator is the $\sin^2 2\theta_{13}$ sensitivity limit defined as the largest fit value which fits the simulated value $\sin^2 2\theta_{13} = 0$. Because the simulated value is computed for $\sin^2 2\theta_{13} = 0$, this sensitivity limit

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will not depend on the simulated value of $\delta_{\rm CP}$, as well as it includes all correlations and degeneracies in a straightforward way. In addition, it can be shown that it will not depend on the mass hierarchy [7], either. As performance indicator, it can serve for the comparison of the potential of different experiments to extract $\sin^2 2\theta_{13}$ from the appearance information, as well as it can be used for risk minimization with respect to Δm^2_{31} or optimization. Similar indicators can be defined for the mass hierarchy and $\delta_{\rm CP}$, where we will focus on one particular quantity in the rest of this talk: The precision of the leptonic CP phase $\delta_{\rm CP}$.

Compared to CP violation measurements, CP precision measurements do not assume that some values of $\delta_{\rm CP}$ are "special", such as the CP conserving values 0 or π, or the maximally CP violating values $\pi/2$ or $3\pi/2$. From theory, we know that there has to be some CP violation at high energies in order to create the baryon asymmetry. However, there is no evidence that this CP violation is connected to the low energy leptonic Dirac CP phase, which means that from theory there is now general argument why certain values of $\delta_{\rm CP}$ should be realized by nature. Thus, we discuss here the more general question of the precision of the measurement of $\delta_{\rm CP}$ as the most sophisticated measurement in neutrino oscillation physics. In particular, for superbeams, where CP violation measurements are very difficult, but nevertheless some values of $\delta_{\rm CP}$ might be excluded, a new performance indicator is needed. Therefore, we define the "CP coverage" [2] as the range of fit values of $\delta_{\rm CP}$ which fit a certain true value. Hence, a very small CP coverage corresponds to the precision of the measurement, whereas a CP coverage close to 360° means that no information on $\delta_{\rm CP}$ can be obtained. For example, a CP coverage of 300° implies that no CP violation measurement is possible, but 60° of all possible values for $\delta_{\rm CP}$ can be excluded. Eventually, the concept of the CP coverage can be used to evaluate the performance of next-generation beam experiments, as well as of future high precision instruments.

CP precision measurements are strongly influenced by the simulated values of $\sin^2 2\theta_{13}$ and $\delta_{\rm CP}$ itself (besides other parameters). Two performance indicators are especially useful in this context: The CP coverage as function of the simulated value of $\sin^2 2\theta_{13}$ ("CP scaling", for fixed simulated value of $\delta_{\rm CP}$), and the CP coverage as function of the simulated value of $\delta_{\rm CP}$ ("CP pattern", for fixed simulated value of $\sin^2 2\theta_{13}$). The dependence on $\sin^2 2\theta_{13}$ (CP scaling) essentially depends (to a first approximation) on the event numbers in the appearance rates and is related to the experiment performance. For example, superbeams do have a different $\sin^2 2\theta_{13}$-range, where they return useful results, than neutrino factories (for an overview, see Ref. [8]). Therefore, CP scalings are good performance indicators to compare different classes of experiments. The dependence on $\delta_{\rm CP}$ (CP pattern), however, is related to the intrinsic structure of the oscillation probabilities and can in simple cases be interpreted in terms of bi-rate graphs [4,9]. It can be used for risk minimization with respect to the unknown true value of $\delta_{\rm CP}$.

The computation of the CP coverage for a fixed given set of simulated parameter values is, in principle, rather straightforward: For any degeneracy as well as the best-fit manifold, the fit manifold is projected onto the $\delta_{\rm CP}$-axis. The CP coverage can then be read off as the fraction of all possible CP values which fit the chosen simulated value. However, especially for neutrino factories, the computation as well as interpretation of CP patterns and scalings becomes very complicated for several reasons:

1. Neutrino factories have rather good spectral information, which means that there is no simple interpretation of the CP patterns in terms of bi-rate graphs.

2. The $(\delta_{\rm CP}, \theta_{13})$-degeneracy [3] causes a strong non-Gaussian dependence on the confidence level.

3. The $\mathrm{sgn}(\Delta m^2_{31})$-degeneracy [4] starts moving for small values below $\sin^2 2\theta_{13} \sim 10^{-3}$ (*cf.*, Fig. 8 of Ref. [2]).

4. Matter density uncertainties become relevant for large values of $\sin^2 2\theta_{13}$ [3,10].

5. The topology of the fit parameter space becomes very flat below $\sin^2 2\theta_{13} \sim 10^{-4}$, *i.e.*,

it contains lots of local minima at small χ^2-values [11].

Therefore, the CP coverage is not at all a trivial quantity for neutrino factories, since it contains a lot of highly condensed information. One can even define more condensed performance indicators, such as a conservative case CP scaling, where one does not choose one fixed simulated value for δ_{CP}, but computes the conservative case over all values. Such an indicator can be found in Fig. 19 of Ref. [2]. The computation of this indicator, however, is very sophisticated and requires a lot of computation time.

In summary, the treatment of existing experiments is rather straightforward: One simply shows all allowed fit regions (degeneracies) to the data. For future experiments, however, the data have to be simulated, which implies that the results depend on the simulated parameter values. Therefore, one actually faces a 12-dimensional parameter space. We conclude that it is hence necessary that one

- identify the most relevant impact simulated parameters.
- identify the purpose of the evaluation (such as optimization, risk minimization, comparison of experiments *etc.*).
- condense the information by the definition of appropriate performance indicators.
- present the performance indicators as function of the relevant impact parameters within their currently allowed ranges.

One such very sophisticated performance indicator is the CP coverage, which not only describes the exclusion of different δ_{CP}-values by next-generation experiments, but also the precision at future high-performance instruments. We have demonstrated several of the complications of the computation and interpretation of CP coverage measurements. Especially the analysis of neutrino factories has turned out to be very sophisticated if one wants to identify all of the relevant parameter space. We conclude that the path leading to future high-precision neutrino oscillation measurements such as β-Beams [12,13] or neutrino factories [14] needs further illumination with respect to the "complete" fit and simulated parameter space. This also involves the discussion of conceptual alternatives, such as the "silver channels" [15], the "magic baseline" option [16], or the combination with superbeam upgrades [17].

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